Second mini-workshop on Differential Equations and Dynamical Systems

Working on some recent trends

Foz do Arelho, April 20-22, 2022

How to reach Foz do Arelho

(credits to Paolo Gidoni)

There is no direct connection between Lisbon and Foz do Arelho. The nearby city of **Caldas da Rainha** (about 10km far from FdA) is well connected and from there one can continue by bus or taxi.

- *From Lisbon to Caldas da Rainha*

The best way to reach Caldas da Rainha from Lisbon is **by bus**. The service is offered by two companies:

1. **Rede-expressos** (https://www.rede-expressos.pt)
2. **Rodoviaria do Oeste** (http://rodoviariadooeste.pt)

Both bus terminals can be reached by metro from the airport in about 40m (15m from the city center).

1. The buses of **Rede-Expressos** leave from the bus terminal of **Lisboa-Sete Rios** (near the **Jardim Zoologico metro station** on the **blue line**). A one-way trip takes about 1h10m and should cost approximately 9 euros. Tickets can also be bought online up to a month before departure.

Buses should leave from Lisbon (April 19/20) at 7.00, 7.45, 8.30, 9.00, 10.00, 10.30, 11.00, 12.00, 13.00, 14.00, 14.30, 15.45, 16.30, 17.00, 17.15, 18.15, 19.00, 20.00, 21.15, 22.30 (please **check on the website**)

Buses should leave from CdR (April 22) at 7.20, 7.30, 8.35, 9.35, 10.00, 10.30, 10.50, 11.15, 11.50, 13.05, 13.50, 14.20, 15.00, 15.30, 15.45, 16.35, 17.50, 18.30, 18.50, 19.20, 20.00, 20.20, 20.50, 20.55, 22.05 (please **check on the website**)

2. The buses of **Rodoviaria do Oeste** (**"rapida verde"** line) leave from the bus terminal of **Lisboa-Campo Grande** (metro station **Campo Grande** on the **yellow** or the **green line**). A one-way trip takes about 1h15m and should cost approximately 8.10 euros.

The full schedule can be found here:
http://rodoviariadooeste.pt/wp-content/uploads/rapida_linha_verde.pdf

It is also possible to travel from Lisbon to CdR by **train**; however there is no direct connection, trips are longer (around 2h30m) and less frequent.

- *From Caldas da Rainha to Foz do Arelho*

There are a **few buses** between CdR and FdA operated by **Rodoviaria do Oeste**. A one-way trip takes about 24m.

Taxis are also available, both in Caldas da Rainha and Foz do Arelho. The cost of one trip should be around 12 euros+extras.

Schedule of the Workshop

Venue: Espaço José Malhoa

Wednesday - April 20th		Thursday - April 21st		Friday - April 22nd	
Opening	09:00-09:10	**I. Coelho**	09:00-09:40	**C. Rebelo**	09:00-09:40
R. Ortega	09:10-10:00	**E. Sovrano**	09:40-10:20	**G. Feltrin**	09:40-10:20
V. Ortega	10:10-10:40	**C. Silva**	10:20-11:00	**A. Tellini**	10:20-11:00
Break	*10:40-11:00*	*Break*	*11:00-11:20*	*Break*	*11:00-11:20*
W. Dambrosio	11:00-11:40	**P. Gidoni**	11:20-12:00	**S. Mosconi**	11:20-12:00
J. Bellver	11:40-12:20	**A. Margheri**	12:00-12:40	**P. Torres**	12:00-12:40
M. Garrione	12:20-13:00			***Closing***	12:40-12:50
Lunch	*13:00-14:30*	*Lunch*	*12:40-14:30*		
Working Groups	14:30-19:00	***Working Groups***	14:30-19:00		

Invited Seminars

Vector-borne disease outbreak control via instant vector releases

Jesús BELLVER ARNAU
Laboratoire Jacques Louis Lions - Sorbonne Université & INRIA-Paris, France

Vector-borne diseases have a significant impact on human health worldwide, accounting for 17% of all infectious diseases. These diseases can be caused by parasites, bacteria, or viruses and are transmitted by different types of vectors, such as ticks, fleas, or mosquitoes.
In this work, we present the study of optimal vector release strategies to control vector-borne diseases such as dengue, Zika, chikungunya and malaria. Two techniques are considered:

- The sterile insect technique (SIT), which involves releasing sterilized male vectors among wild vectors to disrupt their reproduction.

- The use of Wolbachia (currently used primarily for mosquitoes), which consists of releasing vectors infected with a bacterium that limits their ability to transmit the pathogen in order to replace the wild population.

In each case, we consider an epidemiological model including humans and vectors, the temporal dynamics of the populations is modeled by a system of ordinary differential equations. Nevertheless, to simplify the study and given that the duration of the releases is very small compared to the duration of the outbreak, the releases are considered instantaneous. Mathematically they are represented by linear combinations of Dirac measures with positive coefficients determining their intensity.
We model the question of the optimal mosquito release strategy by an optimal control problem that we solve numerically using ad hoc algorithms, based on the writing of first order optimality conditions characterizing the best combination of Dirac measures. The objective expressed by the cost function is here to reduce in a fixed time horizon the amount of human infections.
We then discuss the results obtained, focusing in particular on the complexity and efficiency of the optimal controls and comparing the strategies obtained for the two techniques.

Seasonally dependent competitive Kolmogorov systems: extinction or coexistence?

Isabel COELHO
Instituto Superior de Engenharia de Lisboa & CEMAT at FC-ULisboa, Portugal

We will consider a periodic Kolmogorov system describing the interaction of two competing species experiencing a seasonally fluctuating environment. Under assumptions that generalise Gopalsamy conditions, we will discuss coexistence or extinction of one or both species, and describe the domain of attraction of the nontrivial periodic solutions in the axes.
Our results may be applied to models with nonlinear competition, which have been studied in the biological literature, such as models of microbial growth or of phytoplankton competition under the effect of toxins.
Time permitting, we will also consider a two species amensalism model, a biological interaction in which one species may harm the other, but the reverse is not true.

Joint work with Carlota Rebelo (Universidade de Lisboa) and Elisa Sovrano (Università di Modena e Reggio Emilia)

Unbounded solutions to a system of coupled asymmetric oscillators at resonance

Walter DAMBROSIO
Dipartimento di Matematica "G. Peano", Università degli Studi di Torino, Italy

In this seminar we present a result on the existence of unbounded solutions for a coupled asymmetric oscillator of the form

$$\begin{cases} \ddot{x}_1 + a_1 x_1^+ - b_1 x_1^- + \phi_1(x_2) = p_1(t) \\ \ddot{x}_2 + a_2\, x_2^+ - b_2\, x_2^- + \phi_2(x_1) = p_2(t), \end{cases}$$

where $\phi_i : \mathbb{R} \to \mathbb{R}$ is locally Lipschitz continuous and bounded, $p_i : \mathbb{R} \to \mathbb{R}$ is continuous and 2π-periodic and the positive real numbers a_i, b_i satisfy

$$\frac{1}{\sqrt{a_i}} + \frac{1}{\sqrt{b_i}} = \frac{2}{n}, \qquad \text{for some } n \in \mathbb{N}.$$

The proof relies on a careful investigation of the dynamics of the (four-dimensional) Poincaré map associated to the system, using action-angle coordinates and a discretization approach.

Joint work with Alberto Boscaggin (Università di Torino) and Duccio Papini (Università di Udine)

Periodic perturbations of central force problems and an application to a restricted 3-body problem

Guglielmo FELTRIN
University of Udine, Italy

In this talk, we deal with a perturbed central force problem of the form

$$\ddot{x} = V'(|x|)\frac{x}{|x|} + \varepsilon\,\nabla_x U(t,x), \quad x \in \mathbb{R}^2 \setminus \{0\},$$

where $\varepsilon \in \mathbb{R}$ is a small parameter, $V\colon (0,+\infty) \to \mathbb{R}$ and $U\colon \mathbb{R} \times (\mathbb{R}^2 \setminus \{0\}) \to \mathbb{R}$ are smooth functions, and U is τ-periodic in the first variable. Based on the introduction of suitable time-maps (the radial period and the apsidal angle) for the unperturbed problem ($\varepsilon = 0$) and of an associated non-degeneracy condition, we apply a higher-dimensional version of the Poincaré–Birkhoff fixed point theorem to prove the existence of non-circular τ-periodic solutions bifurcating from invariant tori at $\varepsilon = 0$. Our new non-degeneracy condition is nothing but an equivalent formulation of the usual one, having however the advantage of depending only on the potential V and, instead, not requiring the explicit knowledge of the Hamiltonian in action-angle coordinates.
Next, we show that this non-degeneracy condition is satisfied for some concrete examples of physical interest (including the homogeneous potential $V(r) = \kappa/r^\alpha$ for $\alpha \in (-\infty, 2)\setminus\{-2, 0, 1\}$). At last, an application is given to a restricted 3-body problem with a non-Newtonian interaction.

Joint work with Alberto Boscaggin and Walter Dambrosio (Università di Torino)

Symmetry and stability in some nonlinear beam equations

Maurizio GARRIONE
Department of Mathematics, Politecnico di Milano, Italy

We consider a hinged beam with a finite number r of internal constraints, whose vertical displacement $u = u(x,t)$ ($x \in I = [-\pi, \pi]$, $t \in \mathbb{R}$) obeys the fourth-order evolution equation

$$u_{tt} + u_{xxxx} + \|u\|_{L^2(I)} u = 0$$

and the boundary-internal conditions $u(-\pi, t) = u(\pi, t) = u(\kappa_i \pi, t) = 0$, with $\kappa_i \in (-1, 1)$ for every $i = 1, \dots, r$, $\kappa_i \neq \kappa_j$ for $i \neq j$. We wonder whether a symmetric configuration of the internal constraints is convenient or not in terms of the stability of the solutions. As an idea, we deal with bi-modal solutions

$$u(x,t) = \varphi(t) e_\lambda(x) + \psi(t) e_\rho(x),$$

where e_λ and e_ρ are two fixed eigenfunctions of the associated linear stationary problem and we assume that, for $t = 0$, φ is *prevailing* (large) and ψ is *residual* (small). We aim to enhance, as much as possible in dependence on the position of the constraints, the amplitude threshold of φ beyond which a significant energy transfer takes place towards ψ. Repeating the argument on suitable couples of eigenfunctions, we define an energy threshold of instability for the considered beam, and we seek the configuration of the internal constraints which maximizes it. For the considered equation, such a problem is related to the spectral properties of the associated linear stationary problem, which we briefly recall together with some related questions to be further investigated.

Joint work with Filippo Gazzola (Politecnico di Milano)

Asymptotic stability of running-periodic solutions in some dynamic models of crawling locomotion

Paolo GIDONI
Czech Academy of Sciences, Prague
Czech Republic

Alessandro MARGHERI
University of Lisbon
Portugal

Biological and bio-inspired locomotion is usually described by recognizing periodic patterns, or gaits, in the movement of limbs or other body parts: the flapping of a fin for a fish, the movement of the legs during a stride for a walking o running animal, the peristaltic wave for an earthworm.
But is the evolution of the system actually periodic? Or more properly, running-perioric (or derivo-periodic), since, ideally, each cycle will propel the animal (or robot) a little bit forward? The answer is usually no, due to inertia and elasticity. However, often we might expect the behaviour to converge asymptotically to a running-periodic one. In this double talk, we will rigorously study such asymptotic behaviour for some dynamic models of crawling locomotion.
In Part I , we will introduce the problem, discuss its relevance also for other issues such as optimal control, describe the models of crawling locomotion studied in Part II, and illustrate with examples some additional questions, such as the lack of uniqueness (up to symmetries) of running periodic solutions, which might lead to the same gait producing different asymptotic velocities for different initial conditions.
In Part II, we will present some results concerning the asymptotic stability and uniqueness of running-periodic solution for some dynamic models, whose evolution can be described respectively by a special class of first order multivalued ODEs or by a system of second order ODEs. We will also discuss how, in the case of dry friction, we might observe both finite-time convergence and asymptotic-only convergence to a running-periodic solution.

Joint work with Carlota Rebelo (Universidade de Lisboa)

Uniqueness of maximum point for the p-torsion function of convex domains in the plane

Sunra J. N. MOSCONI
Department of Mathematics and Computer Sciences, University of Catania, Italy

Given a convex domain Ω in the plane, we will consider the unique solution of the so-called *Torsion problem*, i. e.

$$\begin{cases} -\Delta_p u = 1 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

Here $p > 1$, $-\Delta_p = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian operator and no regularity is assumed on $\partial\Omega$ beyond the natural Lipschitz one coming from convexity.
It is well-known from [4] that the *torsion function* u solving the problem is log-concave, meaning that $\log u$ is concave. When $p = 2$ and in any dimension, $\log u$ is also strictly concave, thus it has a unique maximum point, a property inherited by u itself. This is usually proved through the *Constant Rank Principle*, or *Microscopic Convexity Principle* of Caffarelli-Friedman [2, 3], ensuring that the rank of $D^2 \log u$ is constant on Ω. Due to the singular/degenerate nature of the p-Laplacian, the Constant Rank Principle (whenever it can be stated) is no longer true when $p \neq 2$, and different techniques are needed.
By analising the *heart* of the convex body Ω (see [1]) and through the Aleksandrov reflection method, we are able to prove that u attains its maximum at a unique point of Ω. As a byproduct, we derive the strict concavity of $\log u$, hence the strict convexity of the super-level sets of u, even if the domain Ω has large flat parts, as in the case of a square.
The arguments are essentially geometric, relying on the existence of shadow-sections of convex bodies which only holds in dimension two.
If time permits, we will briefly discuss a similar statement for solutions of $-\Delta_p u = f(u)$ with suitable reactions f and an example showing that Aleksandrov technique has little hope to succeed in dimension greater or equal than three.

Joint work with William Borrelli and Marco Squassina (Università Cattolica del Sacro Cuore - Brescia)

References

[1] L. Brasco and R. Magnanini, The heart of a convex body. In: Geometric properties for parabolic and elliptic PDE's, 49–66, Springer INdAM Ser., 2, Springer, Milan, 2013.

[2] L. A. Caffarelli and A. Friedman, Convexity of solutions of semilinear elliptic equations, Duke Math. J. 52 (1985), 431–456.

[3] N. J. Korevaar and J. L. Lewis, Convex solutions of certain elliptic equations have constant rank Hessians, Arch. Rational Mech. Anal. 97 (1987), 19–32.

[4] S. Sakaguchi, Concavity properties of solutions to some degenerate quasilinear elliptic Dirichlet problems, Ann. Scuola Norm. Sup. Pisa Cl. Sci. 14 (1987), 403–421.

A regularized variational principle

Rafael ORTEGA
Departamento de Matemática Aplicada, Universidad de Granada, Spain

Consider the perturbed Kepler problem

$$\ddot{x} = -\frac{x}{|x|^3} + \nabla_x U(t,x), \quad x \in \mathbb{R}^2 \tag{1}$$

where U is smooth and T-periodic in t. The action functional

$$\mathcal{A}(x) = \int_0^T \{\frac{1}{2}|\dot{x}|^2 + \frac{1}{|x|} + U(t,x)\}dt, \quad x = x(t),\ T - \text{periodic}$$

is well defined for functions x without collisions. The critical points of $\mathcal{A}$ are the classical periodic solutions of the equation. The change of variables

$$x = z^2, \quad ds = \frac{dt}{|x|}$$

is usually employed to regularize the equation (Levi-Civita). We show that this change is also useful to regularize the functional. The critical points of the new functional are in correspondence with the generalized periodic solutions.

Joint work with Vivina Barutello (Università di Torino) and Gianmaria Verzini (Politecnico di Milano)

Periodic and quasi-periodic orbits around a periodically perturbed point-vortex

Víctor ORTEGA
University of Granada, Spain & Universidade de Lisboa, Portugal

Let us consider a model of a periodic Hamiltonian system in the plane with a singularity: a point-vortex under a periodic perturbation.
In a perfect fluid, a point-vortex is essentially a singularity of the vorticity, and can be modeled by the Hamiltonian $\Psi_0(x, y) = \frac{1}{2}\ln(x^2 + y^2)$ being x and y the usual rectangular coordinates in the plane. The associated system is integrable and the solutions are periodic, rotating around the vortex in circular paths.
If we introduce an external periodic perturbation $p(t, x, y)$, the perturbed system ideally models the passive transport of particles in a fluid subjected to the action of a steady vortex placed at the origin and an external time-dependent background flow. Our result gives sufficient conditions on $p(t, x, y)$ for the existence of periodic and quasi-periodic solutions. To prove our result, we will apply a suitable version of the Aubry-Mather theory to the Poincaré map associated to the perturbed system.

Joint work with Stefano Marò (Università di Pisa)

References

[1] S. Marò and V. Ortega, Twist dynamics and Aubry-Mather sets around a periodically perturbed point-vortex, J. Differential Equations 269 (2020), 3624–3651

On a SIS model with heterogeneity in susceptibility

Carlota REBELO
Departamento de Matemática and CEMAT, FCUL, Universidade de Lisboa, Portugal

The hypothesis that infection prevalence in a population correlates negatively with variance in the susceptibility of its individuals has support from experimental, field, and theoretical studies. However, its generality has never been formally proved. Here we formulate an endemic SIS model with individual susceptibility distributed according to a discrete or continuous probability function to assess the generality of such hypothesis.

Joint work with M. Gabriela M. Gomes (University of Strathclyde and Universidade de Porto) and Alessandro Margheri (Universidade de Lisboa)

Admissibility and μ-dichotomies with respect to the sequences of norms

César M. SILVA
Departamento de Matemática, Universidade da Beira Interior, Covilhã, Portugal

Abstract: Let $(X, \|\cdot\|)$ be a Banach space and $(A_n)_{n\in\mathbb{N}}$ be a sequence of bounded linear operators acting on X. Denote the *discrete evolution family* associated with the sequence $(A_n)_{n\in\mathbb{N}}$ by $\mathcal{A} = (\mathcal{A}_{m,n})_{m\geqslant n}$. Let $(\mu_m)_{m\in\mathbb{N}}$ be a strictly increasing sequence of nonnegative numbers that converges to $+\infty$ and let $(\|\cdot\|_m)_{m\in\mathbb{N}}$ be a sequence of norms in X such that, for each fixed m, the norm $\|\cdot\|_m$ is equivalent to $\|\cdot\|$. We introduce in this work the following notion: we say that the sequence of linear operators $(A_m)_{m\in\mathbb{N}}$ admits a *μ-dichotomy with respect to the sequence of norms* if there are projections P_m, $m \in \mathbb{N}$, such that $A_m \,|\ker P_m \to \ker P_{m+1}$ is invertible, $P_m\mathcal{A}_{m,n} = \mathcal{A}_{m,n}P_n$, $m, n \in \mathbb{N}$, and there are constants $\lambda, D > 0$ such that, for every $x \in X$ and $n, m \in \mathbb{N}$, we have

$$\|\mathcal{A}_{m,n}P_n x\|_m \leqslant D(\mu_m/\mu_n)^{-\lambda}\|x\|_n, \quad \text{for } m \geqslant n$$

$$\|\mathcal{A}_{m,n}Q_n x\|_m \leqslant D(\mu_n/\mu_m)^{-\lambda}\|x\|_n, \quad \text{for } m \leqslant n$$

where $Q_m = \mathrm{Id} - P_m$ is the complementary projection and, for $m \leqslant n$, we use the notation $\mathcal{A}_{m,n} = (\mathcal{A}_{n,m})^{-1} : \ker P_n \to \ker P_m$.
The objective of this talk it to obtain characterizations of μ-dichotomies with respect to sequences of norms based on admissibility conditions. Additionally, we use the obtained characterizations to derive robustness results for the considered dichotomies. As particular cases, we recover several results in the literature concerning nonuniform exponential dichotomies and nonuniform polynomial dichotomies [1, 2]. We also obtain new results for nonuniform dichotomies with logarithmic growth. This talk is based on [3].

References

[1] L. Barreira, D. Dragičević and C. Valls, Nonuniform hyperbolicity and one-sided admissibility, Rend. Lincei Mat. Appl. 27 (2016), 1–13.

[2] D. Dragičević, Admissibility and nonuniform polynomial dichotomies, Math. Nachr. 93 (2019), 226–243.

[3] C.M. Silva, Admissibility and generalized nonuniform dichotomies for discrete dynamics, Commun. Pure Appl. Anal., accepted for publication.

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Reaction-convection equations with non-monotone diffusion: what are the consequences of this diffusion on wavefronts?

Elisa SOVRANO
Università degli Studi di Modena e Reggio Emilia, Italy

We consider a class of reaction-convection equations with a degenerate diffusion term driven by a nonlinear, bounded, and non-monotone function (of the gradient) that approaches zero at infinity. The Perona-Malik operator provides a paradigmatic example of this type of diffusion playing an essential role in image processing. When the reaction term is monostable, we give a complete picture of regular monotone wavefronts between two steady states in terms of their wave speed. We also show that the only possible regular wavefronts are those whose slope never exceeds the critical values of the diffusion.

Joint work with Andrea Corli (Università di Ferrara) and Luisa Malaguti (Università di Modena e Reggio Emilia)

Multiplicity of nodal steady-states for classical logistic equations

Andrea TELLINI
Universidad Politécnica de Madrid, Spain

Using as a base the result on multiplicity of 1-node solutions for degenerate logistic equations (cf. [2]), we will show how mutiplicity also occurs for classical logistic equations. This result is obtained in [1] for weights which are small perturbations of the weights of the degenerate case, although it is important to point out that the range of parameters for which multiplicity holds are different.
In addition, we will also present some numerical simulations which suggest that multiplicity also holds for weights which are not small perturbations of the degenerate ones, and actually are quite close to situations where there is uniqueness.

Joint work with Pablo Cubillos and Julián López-Gómez (Universidad Complutense de Madrid)

References

[1] P. Cubillos, J. López-Gómez and A. Tellini, Multiplicity of nodal solutions in classical non-degenerate logistic equations, Electronic Research Archive 30 (2022), 898-928.

[2] J. López-Gómez and P. Rabinowitz, The structure of the set of 1-node solutions of a class of degenerate BVP's, J. Differential Equations 268 (2020), 4691–4732.

Periodic solutions of the Lorentz force equation

Pedro J. TORRES
University of Granada, Spain

The Lorentz force equation rules the dynamics of a charged particle under the action of an electromagnetic field. In this talk, we review some recent results about the presence of periodic motions where the electromagnetic field is time-periodic. Some open problems are identified.